Integration Technique - Trigonometry

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Questions in past papers often come up combined with other topics.
Topic tags have been given for each question to enable you to know if you can do the question or whether you need to wait to cover the additional topic(s).

Scan the QR code(s) or click the link for instant detailed model solutions!

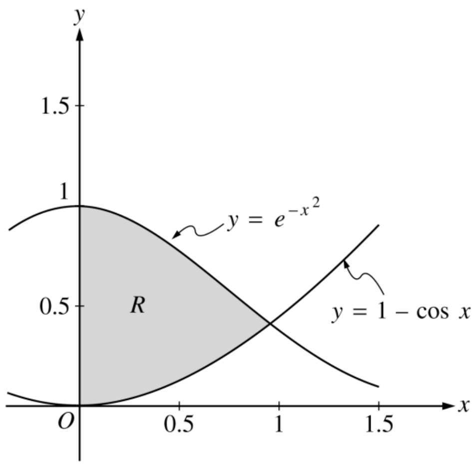
Question 1

Qualification: AP Calculus AB

Areas: Applications of Integration

Subtopics: Volume using Cross Sections, Area Between Curves, Volume of Revolution – Washer Method, Integration Technique – Trigonometry, Integration Technique – Exponentials

Paper: Part A-Calc / Series: 2000 / Difficulty: Easy / Question Number: 1



1. Let R be the shaded region in the first quadrant enclosed by the graphs of $y = e^{-x^2}$, $y = 1 - \cos x$, and the y-axis, as shown in the figure above.

   (a) Find the area of the region R.

   (b) Find the volume of the solid generated when the region R is revolved about the x-axis.

   (c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of this solid.

Question 2

Qualification: AP Calculus AB

Areas: Differential Equations

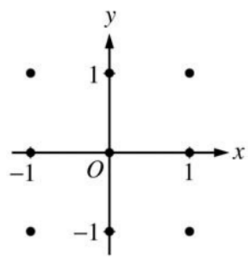
Subtopics: Sketching Slope Field, Integration Technique - Harder Powers, Integration Technique – Trigonometry, Separation of Variables in Differential Equation, Initial Conditions in Differential Equation

Paper: Part B-Non-Calc / Series: 2006-Form-B / Difficulty: Easy / Question Number: 5

5. Consider the differential equation $\frac{dy}{dx} = (y-1)^2 \cos(\pi x)$.

(a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.

(Note: Use the axes provided in the exam booklet.)



(b) There is a horizontal line with equation $y = c$ that satisfies this differential equation. Find the value of c.

(c) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(1) = 0$.

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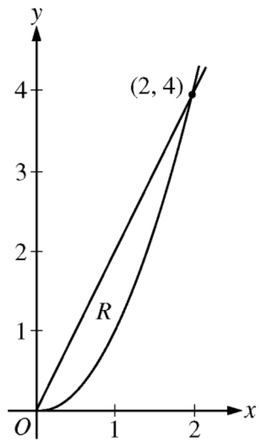
Question 3

Qualification: AP Calculus AB

Areas: Applications of Integration

Subtopics: Area Between Curves, Volume using Cross Sections, Integration Technique – Trigonometry, Integration Technique – Standard Functions

Paper: Part B-Non-Calc / Series: 2009 / Difficulty: Easy / Question Number: 4



4. Let R be the region in the first quadrant enclosed by the graphs of $y = 2x$ and $y = x^2$, as shown in the figure above.

(a) Find the area of R.

(b) The region R is the base of a solid. For this solid, at each x the cross section perpendicular to the x-axis has area $A(x) = \sin\left(\frac{\pi}{2}x\right)$. Find the volume of the solid.

(c) Another solid has the same base R. For this solid, the cross sections perpendicular to the y-axis are squares. Write, but do not evaluate, an integral expression for the volume of the solid.

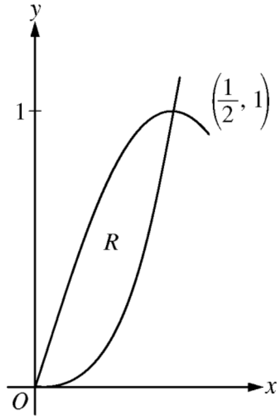
Question 4

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Applications of Integration, Integration

Subtopics: Tangents To Curves, Area Between Curves, Integration Technique – Trigonometry, Volume of Revolution – Washer Method

Paper: Part B-Non-Calc / Series: 2011 / Difficulty: Easy / Question Number: 3



3. Let R be the region in the first quadrant enclosed by the graphs of $f(x) = 8x^3$ and $g(x) = \sin(\pi x)$, as shown in the figure above.

(a) Write an equation for the line tangent to the graph of f at $x = \frac{1}{2}$.

(b) Find the area of R.

(c) Write, but do not evaluate, an integral expression for the volume of the solid generated when R is rotated about the horizontal line $y = 1$.

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Question 5

Qualification: AP Calculus AB

Areas: Limits and Continuity, Applications of Integration, Differentiation

Subtopics: Continuities and Discontinuities, Average Value of a Function, Integration Technique – Exponentials, Integration Technique – Trigonometry, Differentiation Technique – Trigonometry, Differentiation Technique – Exponentials

Paper: Part B-Non-Calc / Series: 2011 / Difficulty: Somewhat Challenging / Question Number: 6

6. Let f be a function defined by $f(x) = \begin{cases} 1 - 2\sin x & \text{for } x \le 0 \\ e^{-4x} & \text{for } x > 0. \end{cases}$

(a) Show that f is continuous at $x = 0$.

(b) For $x \neq 0$, express $f'(x)$ as a piecewise-defined function. Find the value of x for which $f'(x) = -3$.

(c) Find the average value of f on the interval $[-1, 1]$.

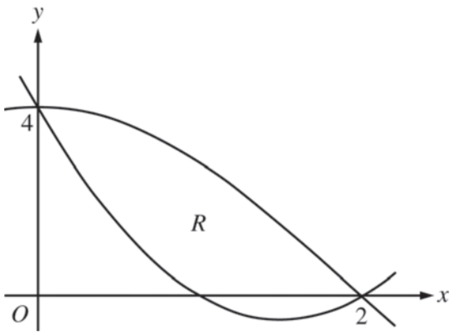
Question 6

Qualification: AP Calculus AB

Areas: Applications of Integration

Subtopics: Area Between Curves, Volume of Revolution – Washer Method, Volume using Cross Sections, Integration Technique – Trigonometry, Integration Technique – Standard Functions

Paper: Part B-Non-Calc / Series: 2013 / Difficulty: Medium / Question Number: 5



5. Let $f(x) = 2x^2 - 6x + 4$ and $g(x) = 4\cos\left(\frac{1}{4}\pi x\right)$. Let R be the region bounded by the graphs of f and g, as shown in the figure above.

(a) Find the area of R.

(b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 4$.

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Write, but do not evaluate, an integral expression that gives the volume of the solid.

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Question 7

Qualification: AP Calculus AB

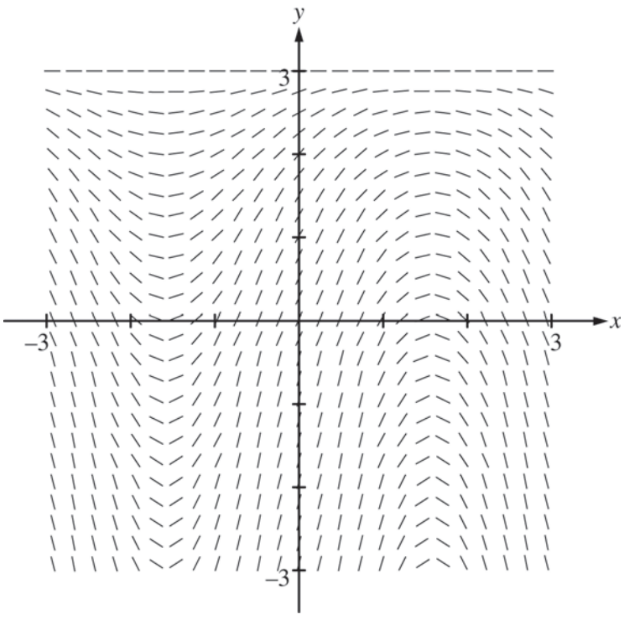
Areas: Applications of Differentiation, Differential Equations

Subtopics: Sketching Slope Field, Tangents To Curves, Particular Solution of Differential Equation, Initial Conditions in Differential Equation, Separation of Variables in Differential Equation, Integration Technique – Trigonometry

Paper: Part B-Non-Calc / Series: 2014 / Difficulty: Medium / Question Number: 6

6. Consider the differential equation $\frac{dy}{dx} = (3 - y)\cos x$. Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(0) = 1$. The function f is defined for all real numbers.

(a) A portion of the slope field of the differential equation is given below. Sketch the solution curve through the point $(0, 1)$.



(b) Write an equation for the line tangent to the solution curve in part (a) at the point $(0, 1)$. Use the equation to approximate $f(0.2)$.

(c) Find $y = f(x)$, the particular solution to the differential equation with the initial condition $f(0) = 1$.

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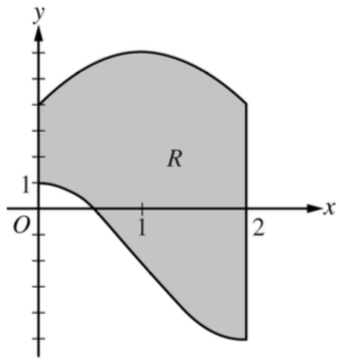
Question 8

Qualification: AP Calculus AB

Areas: Applications of Integration

Subtopics: Area Between Curves, Volume using Cross Sections, Volume of Revolution – Washer Method, Integration Technique – Trigonometry, Integration Technique – Standard Functions

Paper: Part B-Non-Calc / Series: 2019 / Difficulty: Medium / Question Number: 5



5. Let R be the region enclosed by the graphs of $g(x) = -2 + 3\cos\left(\frac{\pi}{2}x\right)$ and $h(x) = 6 - 2(x-1)^2$, the y-axis, and the vertical line $x = 2$, as shown in the figure above.

(a) Find the area of R.

(b) Region R is the base of a solid. For the solid, at each x the cross section perpendicular to the x-axis has area $A(x) = \dfrac{1}{x+3}$. Find the volume of the solid.

(c) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 6$.

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Question 9

Qualification: AP Calculus AB

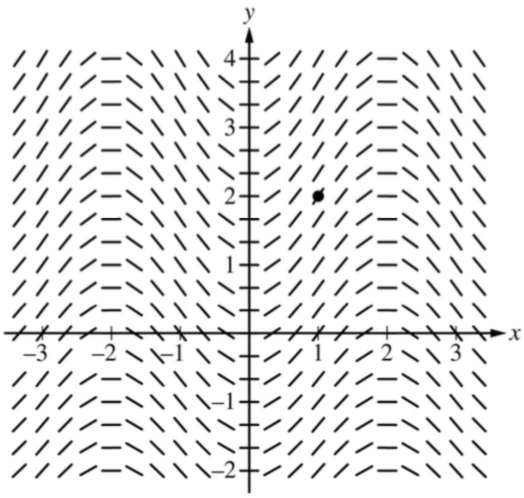
Areas: Differential Equations, Applications of Differentiation

Subtopics: Sketching Slope Field, Tangents To Curves, Separation of Variables in Differential Equation, Particular Solution of Differential Equation, Initial Conditions in Differential Equation, Integration Technique – Trigonometry, Integration Technique - Harder Powers

Paper: Part B-Non-Calc / Series: 2022 / Difficulty: Somewhat Challenging / Question Number: 5

5. Consider the differential equation $\frac{dy}{dx} = \frac{1}{2}\sin\left(\frac{\pi}{2}x\right)\sqrt{y+7}$. Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(1) = 2$. The function f is defined for all real numbers.

(a) A portion of the slope field for the differential equation is given below. Sketch the solution curve through the point $(1, 2)$.



(b) Write an equation for the line tangent to the solution curve in part (a) at the point $(1, 2)$. Use the equation to approximate $f(0.8)$.

(c) It is known that $f''(x) > 0$ for $-1 \le x \le 1$. Is the approximation found in part (b) an overestimate or an underestimate for $f(0.8)$? Give a reason for your answer.

(d) Use separation of variables to find $y = f(x)$, the particular solution to the differential equation $\frac{dy}{dx} = \frac{1}{2}\sin\left(\frac{\pi}{2}x\right)\sqrt{y+7}$ with the initial condition $f(1) = 2$.

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Question 10

Qualification: AP Calculus AB

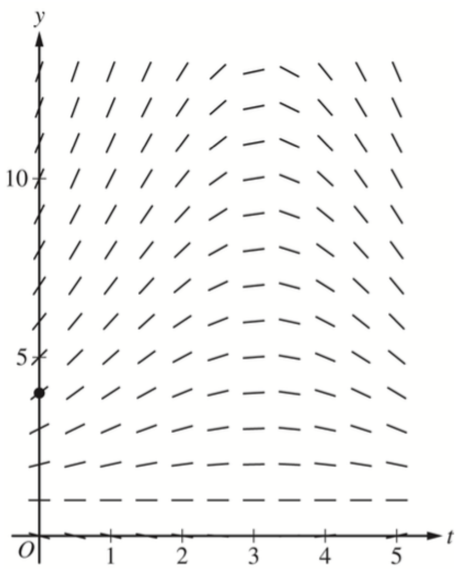
Areas: Differential Equations, Applications of Differentiation

Subtopics: Sketching Slope Field, Local or Relative Minima and Maxima, Separation of Variables in Differential Equation, Initial Conditions in Differential Equation, Particular Solution of Differential Equation, Integration Technique – Trigonometry

Paper: Part B-Non-Calc / Series: 2024 / Difficulty: Medium / Question Number: 3

3. The depth of seawater at a location can be modeled by the function H that satisfies the differential equation $\frac{dH}{dt} = \frac{1}{2}(H-1)\cos\left(\frac{t}{2}\right)$, where $H(t)$ is measured in feet and t is measured in hours after noon ($t = 0$). It is known that $H(0) = 4$.

   (a) A portion of the slope field for the differential equation is provided. Sketch the solution curve, $y = H(t)$, through the point $(0, 4)$.



   (b) For $0 < t < 5$, it can be shown that $H(t) > 1$. Find the value of t, for $0 < t < 5$, at which H has a critical point. Determine whether the critical point corresponds to a relative minimum, a relative maximum, or neither a relative minimum nor a relative maximum of the depth of seawater at the location. Justify your answer.

   (c) Use separation of variables to find $y = H(t)$, the particular solution to the differential equation $\frac{dH}{dt} = \frac{1}{2}(H-1)\cos\left(\frac{t}{2}\right)$ with initial condition $H(0) = 4$.